Configuration Analysis of a Spherical Traction Drive CVT/IVT

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ABSTRACT
Infinitely variable transmission (IVT) characteristics are typically obtained by utilizing a planetary gear set in a split-power transmission configuration. The spherical traction CVT developed by Fallbrook Technologies is kinematically analogous to a variable planetary gear set. The combinations of multiple inputs, outputs, and different internally parallel architectures combine to create hundreds of CVT, IVT, and/or split-power configurations. The variable planetary configuration is inherently power dense and creates a compact, low cost IVT, potentially without the need for dual paths. The control of this novel variator is inherently stable because ratio control is independent of load.

INTRODUCTION
In this paper, we present the analogy between the spherical variator and a conventional planetary gearset. A standard set of nomenclature used to define and analyze this variator is introduced. A simplified review of variator scalability based on the planetary analogy is presented. The planetary analogy also opens the possibility of using the spherical variator in unique powerpaths to achieve the powered zero state characteristic of Infinitely Variable Transmissions (IVTs). In a conventional powersplit transmission, a planetary gearset is used to proportion power flowing through two parallel paths. The same basic layout can be applied to Fallbrook Technologies’ spherical traction CVT, using the CVT itself as the powersplitting device. Four paths have been identified for a basic analysis to illustrate the flexibility of the spherical CVT geometry.

SPHERICAL TRACTION CVT/IVT FUNDAMENTALS
The geometry of the spherical traction CVT patented and under development by Fallbrook Technologies Inc (formerly Motion Technologies LLC) has been presented in previous SAE papers (1,2). A brief review will be conducted here to refresh the reader and present a new standard set of engineering terminology for describing and analyzing the device. A cross section of a simple version of the spherical CVT is shown in Figure 1.

The traction contacts occur at points 1 and 2 between the Input Disk and the Ball, and the Output Disk and Ball, respectively. Referring to Figure 2, ratio is controlled by translation of the idler (Sv component) which, via a cam–follower mechanism, changes the ball rotation axis angle, \( \gamma \). As \( \gamma \) changes, \( r_i \) and \( r_o \) sweep through symmetric ranges which dictates the variator ratio. Ratio control is inherently stable because the ratio is independent of loads at the traction contacts. A stable ratio can be maintained by simple position control of \( \gamma \).

The geometric variables of the spherical variator are illustrated in Figure 2 and defined in section “Spherical CVT Kinematics”. Note that the kinematic diagram in Figure 2 shows a fixed ratio planetary gearset as well, as denoted by the subscript “g” for gear on its variables. This planetary is shown so that we may introduce notation for it in a manner that is differentiated from the similar notation for the variable planetary. The connections shown between the fixed planetary and the variable planetary are for illustration purposes only, and the reader should recognize that there are many ways to connect the two devices, as discussed later in this paper.

The convention presented also differentiates between radial dimensions, which are lowercase (\( r \) for example),...
and notation that refers to physical parts, which is uppercase (Ri is the input disk, for example).

**SPHERICAL CVT KINEMATICS**

![Diagram](image)

**Figure 2: Spherical Variator Nomenclature**

**Variator Component Definitions**
- **Ri** – input disk
- **Ro** – output disk
- **Cv** – cage/carrier (“v” denotes variable)
- **Sv** – Variable sun/idler

**Geometric Variables**
- **γ** – ball tilt angle from horizontal. CCW is positive
- **α** – contact angle from vertical
- **ro** – perpendicular radius from ball axle to output contact
- **ri** – perpendicular radius from ball axle to input contact
- **rc** – carrier theoretical radius (i.e., to ball center)
- **rt** – radius of traction from centerline of transmission to contact patch
- **rb** – ball radius
- **rs** – sun/idler radius
- **rp** – perpendicular radius from ball axle to Sv-ball contact

**Basic equations**

\[
\begin{align*}
ro &= r_b \sin(90 - \alpha - \gamma) \\
ri &= r_b \sin(90 - \alpha + \gamma) \\
rp &= r_b \sin(90 - \gamma)
\end{align*}
\]

To determine component speeds in a spherical CVT, the relative speed equation commonly used to describe speed in a conventional planetary can be used if such losses as creep and spin are neglected for simplicity. In short, the CVT can be treated as two variable planetaries that share a common carrier and sun. Equations 1 & 2 express the speed of Input Disk (Ri), Output Disk (Ro), and Sv relative to Cv. The terms P1 and P2 are train ratios for the Input Disk and the Output Disk. Equations 1 & 2 are a set of two simultaneous equations that can be solved for two unknowns in terms of two knowns. This indicates that for the CVT device to be fully constrained, the speed of two members must be specified. These equations remain constant for all power paths and assume an insignificant amount of creep.

\[
\begin{align*}
\frac{-R_s \cdot R_i}{R_p \cdot R_t} &= -P_1 = \frac{\omega_{R_i} - \omega_{Cv}}{\omega_{Sv} - \omega_{Cv}} \\
\frac{-R_s \cdot R_o}{R_p \cdot R_t} &= -P_2 = \frac{\omega_{R_o} - \omega_{Cv}}{\omega_{Sv} - \omega_{Cv}}
\end{align*}
\]

**SPHERICAL CVT, VARIABLE PLANETARY ANALOGY**

Figure 3 shows the kinematic diagram of a spherical CVT whereby the ball is divided into three planet gears fixed to a common shaft to illustrate the similarity to a compound planetary. Each traction contact noted in Figure 1 has a corresponding planet shown in Figure 3. These planets have a variable diameter, as indicated by the slanted arrows. This figure shows how the spherical CVT is kinematically identical to a fixed ratio planetary geartrain, with variable train ratios. The variability of a train ratio has numerous implications. Note also the convention of “Input side” and “Output side” in this diagram. The “ring gears” of the variable planetary, Ri and Ro, have been thus far referred to as the input disk and output disk. It should be evident that these disks could just as easily be fixed, with the input through either the cage (Cv) or sun (Sv). The implication for the naming convention is that the “Input disk” term should be considered shorthand for “input side disk” since it may not truly be an input. The case is the same for the “output side disk”.

![Diagram](image)

**Figure 3: Kinematic diagram of Spherical CVT**

**Planetary Analogy Implications**

The planetary analogy opens a number of options for using the spherical CVT to sum, divide, and route power from one or more input sources to one or more output
sources. Some examples of where this might find application are:

1. Hybrid vehicles, where the Fallbrook device’s ability to easily accept multiple inputs could replace the fixed ratio planetary in the Toyota hybrid system, for one example of many possible.
2. Power Take-Off (PTO) applications, where the main vehicle drive can be extracted through the variable output, while the PTO comes off of a fixed speed ratio output. This allows variable vehicle speed while maintaining a constant engine power.
3. Infinitely Variable Transmission (IVT) configurations with a powered zero capability and seamless transition from forward to reverse.
4. Wind turbines and other applications requiring a very large ratio change can be accommodated using one of the “infinity” modes that are possible. Real speed ratios have real limitations, as might be expected, but very large ratio changes are not outside the realm of possibility.

Due to this planetary analogy, the term “Continuously Variable Planetary”, or CVP, has been adopted when referring to the spherical CVT.

An additional implication of the planetary analogy lies in the scalability of the CVP. The torque capacity of the CVP increases almost linearly with the addition of extra balls to the variator. A simplified example of how this might apply to an automaker’s lineup is presented in Figure 4 below. If a base single cavity variator (the practicality of single cavity spherical CVT is discussed in our previous paper (2)) is designed to handle the torque of a small, four cylinder vehicle, the same basic parts (input disk, output disk, ball/leg assemblies, idler and control mechanism) can be scaled up in torque capacity by the use of 5, 6, 7 or 8 balls, all in the same transmission case. Only the variable carriers change to accommodate more balls, and even then, the 4 and 8 ball carriers can remain common.

![Figure 4: The CVP can be scaled in torque capacity either by adding balls, by adding another cavity, or both to accommodate a full range of vehicles with a single tooling investment.](image)

Once the torque capacity limits of the single cavity device are reached, the addition of a center output ring, longer shafting, and a mirror image idler control system can leverage the existing components and tooling to create a dual cavity variator. The major tooling investments used to build the input and output disks remain intact, as does ball size and the ball/leg assembly. The transmission case cross section also remains intact, albeit lengthened.

**CVP POWERPATHS**

As discussed earlier, the planetary kinematics of the CVP give rise to many powerpath options for a transmission employing the CVP. Research conducted by the authors has identified more than 1,000 alternatives simply by using the CVP alone, or in conjunction with a planetary gearset on the input, output, or both locations.

The search of the design space used to generate these alternatives is a purely matrix algebra-based approach, which indicates that quite a few of the alternatives are likely either degenerate or mirror image duplicates. However, even after these are culled out, the number of viable combinations remains in the hundreds, and the reader can visualize that even more are possible using compound CVTs and/or compound planetaries.

For the purposes of this paper, four arbitrarily selected powerpaths have been chosen for review. The stick figures for all four powerpaths are shown in Figure 5. Three of the four utilize a fixed ratio planetary gearset in conjunction with the CVP. In Cases 1 and 2, the planetary divides the torque, which is then summed by the CVP, while in Case 3, the CVP divides the input torque, which is then summed by the fixed ratio planetary. Case 4 is an IVT using the CVP only.

In all cases, the reader should note that an external jackshaft is not required to connect the parallel paths. Since the CVP itself can sum or divide power, an external connection is not needed. The absence of an additional jackshaft to connect the parallel powerpaths is a notable characteristic of the CVP architecture. This has obvious advantages in terms of cost and packaging. The fourth powerpath is an IVT using the CVP alone. An IVT without the use of a fixed ratio planetary again has obvious cost and packaging implications.
For powerpaths using a fixed ratio planetary in conjunction with the CVP, another equation is required. The relative motion of the ring and sun gears in terms of the fixed planetary train ratio is related by $P_3$ (see Equation 3).

$$\frac{-R_{sg}}{R_{rg}} = -P_3 = \frac{\omega R_g - \omega C_g}{\omega S_g - \omega C_g} \tag{3}$$

Using Equation 3 with Equation 1 & 2, along with compatibility connection equations, a set of simultaneous equations can be solved to derive the speed ratio equations for each dependent branch of the system. The speed ratio to the output component of the four powerpaths is shown below in Table 1. Reviewing these equations, one can determine IVT capability based upon a difference or sum in the numerator (3). Each speed ratio has been multiplied by a gain value that represents a fixed ratio step-up/down gearbox to modify the speed ratio as a function of $\gamma$.

<table>
<thead>
<tr>
<th>Path</th>
<th>$SR$</th>
<th>IVT?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path 1</td>
<td>$\frac{(P_3 + 1) \cdot P_1 + P_3 - P_2}{P_3 \cdot (P_1 + 1)} \cdot Gain$</td>
<td>No</td>
</tr>
<tr>
<td>Path 2</td>
<td>$\frac{(P_3 + 1) \cdot P_1 - P_2}{P_1 \cdot (P_3 + 1)} \cdot Gain$</td>
<td>Yes</td>
</tr>
<tr>
<td>Path 3</td>
<td>$\frac{P_2 + P_3 \cdot P_1}{P_3 \cdot P_1} \cdot Gain$</td>
<td>No</td>
</tr>
<tr>
<td>Path 4</td>
<td>$\frac{-P_2 + P_1}{P_1} \cdot Gain$</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Figure 5: Kinematic Figures for Powerpaths

Figure 6 shows the speed ratio plotted over a typical range of $\gamma$ for the four powerpaths. The ratio range for each powerpath can be shifted up or down by adjusting the fixed gear planetary train ratio as well as the gain value to create compatible powerpaths or desired ratio ranges.

In Figure 6, powerpath 4 is an IVT using solely the CVP, where powered zero occurs at $\gamma = 0$. Powerpath 2, however, is an IVT where the speed ratio $= 0$ when $\gamma \neq 0$. This is useful when the majority of a duty cycle lies higher than powered zero. Powerpath 2 and similar paths allow one to center the duty cycle of an IVT application closer to $\gamma = 0$ where, among several benefits, efficiency of the device is highest.

To evaluate power flowing through the system, a set of equations is written for both the fixed ratio planetary and the CVP. The equations describing the steady state forces within the fixed ratio planetary are shown below.
Likewise, the equations shown below describe the steady state forces through the CVP.

\[
\begin{align*}
T_{ri} - F_{rib}R_t &= 0 \\
T_{ro} - F_{rob}R_o &= 0 \\
T_{sv} - F_{svb}R_s &= 0 \\
T_{cg} - F_{cvg1} - F_{cvg2} &= 0 \\
F_{rib} + F_{rob} + F_{svb} + F_{cvg1} + F_{cvg2} &= 0 \\
F_{rib}R_i + F_{rob}R_o - F_{svb}R_p &= 0
\end{align*}
\]

Figure 8: CVP Static Force Equilibrium

Using the two sets of equations above and compatibility connection equations, torque in each branch of the device is calculated (3). Figure 9 shows the power ratio between both branches of powerpath 1. As illustrated in Figure 10, the system recirculates power at \( \gamma < 0 \) and switches to split-torque mode for \( \gamma > 0 \).

In Figure 11, power recirculation in powerpath 2 is shown. Figure 12 illustrates how power is flowing as \( \gamma \) sweeps from -20 to 20. Powered zero occurs at \( \gamma = -15 \), where the recirculating power reverses. The device switches from power recirculation to split-torque at \( \gamma = 0 \).

Figure 7: Fixed Ratio Planetary Static Force Equilibrium

Figure 10: Power Flow in Powerpath 2

Figure 11: Powerpath 2 Power Ratio

Figure 12: Power Flow in Powerpath 2
The power ratio for powerpath 3 in Figure 13 shows the split-torque for $\gamma < 0$. At powered zero, where $\gamma = 0$, the system switches to power recirculation for $\gamma > 0$.

![Figure 13: Power Ratio in Powerpath 3](image)

![Figure 14: Power Flow in Powerpath 3](image)

## CONCLUSION

The Continuously Variable Planetary traction drive under development by Fallbrook Technologies, Inc. is kinematically equivalent to a variable planetary gear set and can be configured as an IVT. The power density of the spherical CVT makes it uniquely scalable to a broad range of applications. Dual- or triple-power input (or output) is also possible, with obvious advantages for hybrid vehicles. Combinations of the CVT with a fixed ratio planetary generate numerous powerpaths that can be combined to further increase the ratio range of the device. The transmission components must be capable of handling the recirculating power, which is often far greater than system input power, though IVTs using other types of variators also face this situation.

Since the CVP is based upon a traction drive, it is well suited to high-speed operation with minimal NVH issues as compared to a conventional geartrain or particularly in comparison to chain type CVTs. The well-known disadvantages of a traction device are durability and power capacity, however the power density and use of many contacts to spread load in this design significantly mitigates the severity of these disadvantages.

## REFERENCES